RCMP Research Monograph No. 3

Indices for Measuring the Sustainability and Economic Viability of Farming Systems

Simeon K. Ehui
and
Dunstan S. C. Spencer

Resource and Crop Management Program
International Institute of Tropical Agriculture

1990

The International Institute of Tropical Agriculture (IITA) is an autonomous nonprofit institution, with headquarters on a 1,000-hectare experimental farm at Ibadan, Nigeria. It was established in 1967 as the first major African link in an integrated network of international agricultural research and training centers located in the major developing regions of the world.

Funding for IITA came initially from the Ford and Rockefeller foundations. Land for the experimental farm was allocated by the Government of the Federal Republic of Nigeria. Principal financing has been arranged since 1971 through the Consultative Group on International Agricultural Research (CGIAR).

The Resource and Crop Management Program (RCMP) is concerned with two of the three main thrusts of IITA research, namely: resource management research, which is the study of the natural resource base with a view to refining existing resource management technologies and devising new ones, and crop management research which aims at the synthesis of the products of resource management research and plant breeding into sustainable and productive cropping systems.

The goal of RCMP is to develop economically and ecologically viable farming systems for increased and sustainable production by the smallholder or family farmer of Africa, while conserving the natural resource base.

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Preface

The RCMP Research Monograph series is designed to widely disseminate results of research on the resource and crop management problems of smallholder farmers in sub-Saharan Africa, including socioeconomic and policy-related issues, and to contribute to existing knowledge on improved agricultural principles and policies and the effect they have on the sustainability of small-scale food production systems. These monographs summarize results of studies by IITA researchers and their collaborators; they are generally more substantial in content than journal articles.

The monographs are aimed at scientists and researchers within the national agricultural research systems of Africa, the international research community, policy makers, donors, and international development agencies.

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I. Introduction

Background and objectives

Sub-Saharan Africa (SSA) is the only region of the world where per capita food production has steadily declined over the past two decades. Agricultural output has grown annually by an average of less than 1.5 percent since 1970, with food production rising more slowly than population (IBRD 1989). Although unfavorable farm policies, e.g., inappropriate fiscal and pricing policies, inadequate extension and marketing services, may be responsible in part for the weak agricultural growth, the capability of the natural resource base, especially soils, to sustain continued production under current farming practices is being questioned.

The predominant farming systems in SSA are based on shifting cultivation and related bush fallow systems, with minimal reliance on improved farming inputs. Farmers normally fell and burn the fallow vegetation, cultivate the cleared land for a few (1 to 3) years and then abandon the site for a long period (4 to 20 years) to bush and forest cover. This system is a necessity in the tropics because of the rapid decline in soil productivity under cultivation. Most nutrients are in the biomass. A significant component is recycled and made available to crops when the forest is cleared. Until recently, enough arable land was available to enable use of land in this fashion. Today, however, population growth and socioeconomic changes have resulted in a relative shortage of cultivable land, imposing an excessive demand on the natural resource base. Increased cropping intensities combined with reduced fallow periods have led to further deforestation, soil degradation, and reduced crop yields (OTA 1984b; Matlon and Spencer 1984; Kang et al. 1990).

Most soils of humid tropical Africa are sandy, highly weathered, low in organic matter content, and susceptible to soil erosion and compaction (Lal 1987; El-Ashry and Ram 1987). The challenge faced by decision makers in many nations in SSA is how to feed an increasing population without irreparably damaging the natural resource base on which agricultural production depends (Ehui and Hertel 1989; Ehui et al. 1990).

Clearly, new technologies must be developed which not only increase food production but also maintain ecological stability and preserve the natural resource base, i.e., such technologies must be both economically viable and sustainable (BIFAD 1988). For that reason, the Technical Advisory Committee (TAC) of the Consultative Group on International Agricultural Research (CGIAR) has recommended that research at international centers, which are designed to generate agricultural innovations, should be planned and conducted with a sustainability perspective. There is little guidance available in the literature, however, as to what practical methods are to be used for measuring the sustainability and economic viability of a production system.

Dumanski (1987) critically examined the concept of sustainability as applied to agricultural systems. He concluded that though measurements can focus on soil qualities and on financial viability, current concepts are too broad to be practical, and sustainability is at present difficult to measure. However, Lynam and Herdt (1989) have provided a framework by which the sustainability concept can be empirically incorporated into the research process. They developed a number of propositions, one of which states that "the appropriate measure of output by which to determine sustainability at the crop, cropping or farming system level is total factor productivity (TFP), defined as the total value of all output produced by the system over one cycle divided by the total value of all inputs used by the system over one cycle of the system; a sustainable system has a non-negative trend in TFP over the period of concern".

Building upon that proposition, this paper uses recent advances in productivity measurement and economic index numbers to derive a general approach for measuring economic viability and agricultural sustainability. It is based on the interspatial and intertemporal productivity measures of Denny and Fuss (1983), modified to accommodate changes in natural resource stock and flows. Accounting for natural resource stock in sustainability measurement is particularly important, since a desirable component of sustainable soil and crop management systems in the tropics is a mechanism that can replenish soil nutrients removed by crops, erosion, or leaching, and that can maintain favorable chemical, physical, and biological conditions of the soil under intensive cultivation. In this paper, we concentrate on the measurement of sustainability of farming systems, since our focus is on technologies that are developed for small-scale farmers who produce most of Africa's food and whose farming systems have low and declining productivity (IITA 1990). Also, above the farming system level, so many external factors impact on the sustainability of a system that it is very difficult to determine and separate the source of such impacts (Lynam and Herdt 1989).

Relevance and organization of the study

From a technology evaluation viewpoint, the necessity of measuring sustainability and economic viability is urgent. Too often technologies are developed piecemeal, with little or no regard for sustainability. In Africa, for example, with the exception of relatively limited high potential zones, the stock of new technologies is often most inappropriate, poorly responding to farmers' changing needs and incapable of bringing about a sustainable response in aggregate supply (OTA 1984a; Matlon and Spencer 1984). Also, in the continuing elaboration of criteria, sustainability (in addition to economic viability) has now become the "latest twist by which agricultural development is defined and agricultural technology is evaluated" (Lynam and Herdt 1989). Without proper measurement of these concepts,we cannot make comparisons or assign priorities among competing technologies (CGIAR 1989).

The paper is organized into five sections. In section II, concepts for the measurement of agricultural productivity are discussed. First, traditional approaches to measurement of total factor productivity are reviewed. Second, a model is introduced which takes into account the consequences of changes in natural resource stocks and flows. In section III, measures of agricultural sustainability and economic viability are presented. They are based on the Denny and Fuss (1983) "interspatial" and "intertemporal" total factor productivity measures. In section IV, we consider an empirical example, taking advantage of a unique set of data available at the International Institute of Tropical Agriculture (IITA). Section V contains a summary and some suggestions for future research.

II. Concepts for Measurement of Agricultural Productivity

Traditional productivity measurement

Productivity is generally defined in terms of the efficiency with which the factor inputs (e.g., land, labor, seeds, tools and equipment, fertilizer, etc.,) are converted to output within the production process (Cowing and Stevenson 1981; Capalbo and Antle 1988). There are two measures of productivity: partial productivity and total factor productivity (TFP). The ratio of output to a single input measures the partial productivity of that input. The ratio of output to all inputs combined is the total factor (or multifactor) productivity. As partial productivity relates output to a single input, there are as many partial productivity indices as there are factors of production.

Earlier approaches to productivity measurement were based upon partial productivity indices, typically land or labor productivity. The use of this index-number approach had the advantage of computational simplicity and feasibility, given the general availability of data. Although partial productivity measures provide insights into the efficiency of an input in the production process, they mask many of the factors accounting for observed productivity growth. They are very sensitive to both the composition of outputs and the relative intensity of various inputs. For example, increases over time in an index of output per man-day may simply reflect the substitution of capital for the labor input, the realization of economies of scale, and/or the improvement in labor quality, rather than purely technological progress. For this reason, the use of TFP measures, in which changes in outputs are related to changes in all inputs, is recommended. This will yield a more accurate measure of productivity advancement. Conceptually TFP measures are a clear improvement over single factor measures, since they are based on comprehensive aggregates of outputs and inputs, and changes in the quantity and quality of all inputs can be accounted for (Antle and Capalbo 1988; Cowing and Stevenson 1981; Capalbo and Vo 1988).

Two approaches for measuring TFP have been identified in the literature: (1) the growth accounting (or index-number) approach which involves mainly the development of indices of outputs and inputs and the computation of nonparametric factor productivity measures; and (2) the econometric (or parametric) approach, which is based on econometric estimation of the production function or (through duality relations) the underlying cost or profit function.

In this paper we focus on the nonparametric approach for a number of reasons. First, with the index-number approach, very detailed data with many input and output categories can be used, regardless of the number of observations over time. There are, therefore, no degrees of freedom problems or statistical reliability problems in working with small samples. Second, input-output and input separability assumptions are avoided, since there is no need to aggregate all outputs into a single index or aggregate input data into a small number of categories to make estimation possible. And third, under certain technical and market conditions, the econometric and the index-number approaches are found to be equivalent. Recent advances in growth accounting theory have shown that nonparametric methods do indeed impose an implicit structure on the aggregate production technology (Diewert 1976, 1981; Denny et al. 1981; Ohta 1974).

The growth accounting approach is the conventional approach to the measurement of TFP. The latter is based on the exact index-number approach, in which TFP is measured as the ratio of aggregate output (Q) to aggregate input (X):

$$TFP = Q/X. \tag{1}$$

Using (1) involves compiling detailed accounts of inputs and outputs, and then aggregating them into input and output indices to calculate a TFP index. Important in determining the TFP index is the method by which the raw data are combined into a manageable number of subaggregates and then reaggregated. Earlier approaches to TFP measurement used a Laspeyres weighting system, where base period prices are used as aggregation weights. However, the Laspeyres index has been shown to be inexact, except under conditions of a linear production function in which all inputs are perfect substitutes (Christensen 1975; Diewert 1976). Such an assumption implies that an increase in the relative price of any input would cause discontinuation of its use. If a perfect substitute is available at a lower price, there is no rationale for using the higher priced input. Generally one would not want to select an index-number procedure (such as the Laspeyres) that implies unnecessary restrictive assumptions about the substitutability of inputs. To the extent that the Laspeyres indexing method does not accurately describe the nature of the production process, the results of employing this procedure for measuring productivity change would be incorrect.

A better alternative is to use an index-number that is exact for linear homogeneous flexible functional forms for those aggregator functions that are capable of providing a second order approximation to an arbitrary twice differentiable function (Christensen et al. 1971). The class of indices with this property has been termed "superlative" by Diewert (1976). The most popular superlative index of aggregating outputs and inputs is the Divisia index, which is shown to be exact for the case of transcendental logarithm aggregation (i.e., translog) functions (Diewert 1976, 1981). Conceptually it is considered one of the most defensible methods of aggregation for use in productivity analysis (Denny et al. 1981; Diewert 1976; Hulten 1973). The translog function does not require inputs to be perfect substitutes. If the relative price of an input increases, the producer decreases its use (substituting other inputs) until all marginal productivities are proportional to the new prices. Hence, the prices from both periods enter the Divisia index to represent the marginal productivities in both periods (Christensen 1975).

The Divisia index of aggregate output (Q) and aggregate input (X) is defined in terms of proportionate rates of growth ($\dot{Q}$ and $\dot{X}$) as:

$$\dot{Q} = \sum_j (PjQj/R)\dot{Q}j, \text{ and} \tag{2}$$

$$\dot{X} = \sum_i (W_iX_i/C)\dot{X}_i \tag{3}$$

where Pj and Qj are the price and quantity of output j; $\dot{Q}j$ is the proportionate rate of growth of output j, $R = \sum_j PjQj$ is the total revenue; W_i and X_i denote the price and quantity of input i, $\dot{X}_i$

is the proportionate rate of growth of input i, and $C = \sum_i W_i X_i$ is the total cost. Since (from equation (1)), TFP = Q/X, the proportionate rate of growth of TFP is given by:

$$\dot{TFP} = \dot{Q} - \dot{X} \qquad (4)$$

Equation (4) indicates that the rate of growth of total factor productivity is defined as the rate of growth of aggregate output minus the rate of growth of aggregate input. In other words, total factor productivity indices measure the residual growth in output not accounted for by the growth in factor inputs.

Equations (2) and (3) are computationally difficult to evaluate. Because data are in discrete forms (e.g., annual data), it is important, for empirical purposes, to approximate equations (2)-(4). A convenient discrete approximation is provided by the Tornqvist-Theil quantity index (Christensen and Jorgenson 1970):

$$\begin{aligned} \Delta LnQ &= Ln(Q_t/Q_{t-1}) \\ &= 1/2 \sum_j (S_{jt}+S_{jt-1}) Ln(Q_{jt}/Q_{jt-1}) \end{aligned} \qquad (5)$$

for the output aggregation index, and

$$\begin{aligned} \Delta LnX &= Ln(X_t/X_{t-1}) \\ &= 1/2 \sum_j (S_{it}+S_{it-1}) Ln(X_{it}/X_{it-1}) \end{aligned} \qquad (6)$$

for the input aggregation index.

In equation (5), Q_{jt} denotes the quantity of output Q_j produced in period t, and $S_{jt} = P_{jt}Q_{jt}/\sum_j P_{jt}Q_{jt}$ is the revenue share of output Q_j in total revenue in period t. Similarly in equation (6), X_{it} denotes the quantity of input X_i used in period t and $S_{it} = W_{it}X_{it}/\sum_i W_{it}X_{it}$ is the cost share of input X_i in total cost during period t.

The corresponding discrete approximation to equation (4) is thus provided by:

$$\Delta LnTFP = Ln(TFP_t/TFP_{t-1}) = \Delta LnQ - \Delta LnX \qquad (7)$$

Setting the index to equal 1.0 in a particular year and accumulating the measure according to (7) provides the conventional index of TFP. The discrete Divisia indices (5) and (6) have two important properties. First, they can always be computed, given observations on prices and quantities. Secondly, they converge to the continuous forms (2) and (3) as discrete units of time become small enough.

A generalized model for TFP measurement

Agriculture is a sector which utilizes common property natural resources (e.g., soil nutrients, air, water). The stock of these resources affects the production environment, but it is in many cases beyond the control of the farmer. A case in point is nitrogen, among the most important plant nutrients. It is extremely mobile and drained out of the reach of crop roots during periods of heavy rainfall, or it can be denitrified into the atmosphere when waterlogged soil conditions persist. In the presence of leguminous species, atmospheric nitrogen can also be fixed biologically by the agrosystem, thus contributing to the improvement of the fertility of the soil. Squires and Herrick (1990) have shown that when common property resource stocks are utilized, it is inappropriate for productivity measurement to treat resource stock as a conventional input in the neoclassical production technology. Rather the resource stock is more appropriately specified as a technological constraint. This is because for a given input bundle, increases (decreases) in resource abundance change the production technology, increasing (decreasing) resource flows and crop output.

In addition to the stock, resource flows also affect productivity levels. For example, when the soil is depleted of its nutrients (e.g., via leaching, crop uptake, or runoff), the farmer faces an implicit cost in terms of forgone soil productivity. Conversely, when soil quality is replenished (e.g., via nitrogen fixation), the farmer derives some benefit from the system. Soil productivity is increased without incurring additional costs (e.g., on chemical fertilizer). If these costs and benefits are not accounted for in TFP measurement, the results will be biased.

Using Squires and Herrick's (1990) approach, we derive a general model for TFP measurement. The model is different, however, from theirs in that the contribution of resource flows in productivity measures is separated from crop outputs. Unlike the stock-flow production technology of fisheries where the product is the resource flow, in agriculture the resource flow can serve as output and contribute to total profit (e.g., replenishment of soil quality) or directly contribute to production (e.g., depletion of soil quality).

Assuming that current prices are known, when changes in resource stock levels are positive, the maximization problem can be stated as:

$$\max_{Y_t, Z_t} \pi_t = P_{yt}Y_t + P_{zt}Z_t - G(Y_t, Z_t, W_t, B_t, t) \tag{8}$$

where π_t is a measure of aggregate profit in period t, including all benefits and costs of resource exploitation and B_t is the level of resource abundance in period t. The resource stock here is treated as a technological constraint. Because of the open access nature of the resource, it is assumed that each farmer ignores the effect of his production activities on the resource stock and behaves as if the resource is free, i.e., it has a zero shadow value (Capalbo 1986; Squires and Herrick 1990). Z_t denotes the net resource flow in period t (i.e., $B_{t+1}-B_t$), and it is treated as an output, thus contributing positively to the aggregate profit. Y_t is an index of crop outputs; P_{yt} and P_{zt} are the product and resource flow prices, respectively; $G(\cdot)$ is the variable cost function for the optimal combination of variable inputs, where $\partial G(\cdot)/\partial B<0$ and $\partial G(\cdot)/\partial Z_t > 0$. W_t is a vector of variable input prices; t is the time trend, representing the state of technical knowledge.

When changes in resource abundance levels are negative (i.e., $B_{t+1}-B_t = -Z_t$), Z_t is treated as an input, thus contributing negatively to the aggregate profit. This requires modification of the objective function (8) by replacing the (+) sign before $P_{zt}Z_t$ with a (-) sign, and in this case, $\partial G(\cdot)/\partial Z_t < 0$.

Using the first order conditions of equation (8), development of the continuous time Divisia index by method of the growth accounting approach gives (see Annex A)

$$-\partial LnC/\partial t = [PyY/C]\dot{Y} + [(P_ZZ)/C]\dot{Z} - \Sigma j[(WjXj)/C]\dot{X}j - \dot{B} \qquad (9)$$

where $C = \Sigma jWjXj = P_yY + P_zZ$ = total revenue under constant returns to scale.

When changes in the resource stocks are negative, the productivity index becomes:

$$-\partial LnC/\partial t = [PyY/C]\dot{Y} - [(PzZ)/C]\dot{Z} - \Sigma j[(WjXj)/C]\dot{X}j - \dot{B} \qquad (10)$$

where $C = \Sigma jWjXj + P_ZZ = P_yY$ under constant returns to scale.

Equations (9) and (10) indicate that total factor productivity is measured as the residual after the growth rate of output has been allocated among changes in inputs, resource abundance, and flows. The contributions of the resource flow and stock in the productivity measures are $[(P_ZZ)/C]\dot{Z}$ and $\dot{B}$, respectively. The basic difference between (9) and (10) is that, in the former case, the change in resource stock is assumed positive and the resulting flow is treated as an output. In the latter case, the change in resource stock is assumed to be negative and the resulting flow is treated as a net depletion. We clearly see from (9) and (10) that productivity measures are biased unless variations in the resource stock abundance level, as well as resource flows, are accounted for.

III. Measures of Sustainability and Economic Viability

Agricultural sustainability and economic viability defined

A number of groups and individuals have proposed various definitions for sustainability. TAC, for example, defines a sustainable agricultural system as "the successful management of resources for agriculture to satisfy the changing human needs, while maintaining or enhancing the quality of the environment and conserving natural resources" (CGIAR 1988). Along the same lines, a Committee of the American Society of Agronomy states "A sustainable agriculture is one that, over the long term, enhances environmental quality and the resource base on which agriculture depends; provides for basic human food and fiber needs; is economically viable and enhances the quality of life for farmers and society as a whole" (ASA 1989). A third definition, provided by Conway (1985), says: "sustainability is the ability of a system to maintain productivity in spite of a major disturbance, such as caused by intensive stress or a large perturbation". Lynam and Herdt (1989) define sustainability as "the capacity of a system to maintain output at a level approximately equal to or greater than its historical average, with the approximation determined by its historical level of variability". Hence, they define a sustainable system as "one with a non-negative trend in measured output" and "technology contributes to sustainability if it increases the slope of the trend line".

It is in the light of this last definition that we propose a measure of sustainability, based on the intertemporal TFP measure as developed by Denny and Fuss (1983) which is defined in terms of the productive capacity of a system over time. Intertemporal TFP is an appropriate measure of sustainability as it addresses the question of change in the productivity of a system between two (or more) periods. A system will be said to be sustainable if the associated intertemporal index, which incorporates and values changes in common property resource stock, is not decreasing.

Unlike sustainability, "economic viability" is a static concept which refers to the efficiency with which resources are employed in one system compared with another during one growing season. A new production system can be said to be economically more viable (or efficient) than an existing one if the TFP of the former is higher than that of the latter at a particular point in time. By higher TFP, we mean the capacity of the new technology to produce more output than the existing one after accounting for differences in the input quantities, qualities, and unpriced natural resources used in each system. Alternatively, it can be interpreted as the capacity of the new production system to produce outputs with lower total costs than the traditional one, after accounting for differences in output levels, input prices, unpriced natural resources, and any other state of nature or exogenous variable. Thus, to compare the economic viability of production systems, we advocate the concept of interspatial TFP, which is defined in terms of the productive capacity of a system compared with another over a single period of time, including the unpriced contributions from natural resources to production.

Figure 1 illustrates the difference between intertemporal and interspatial TFP for two hypothetical systems. System 1 is sustainable since its intertemporal TFP increases from a to d over the same period. On the other hand, system 2 is economically more viable than system 1 in year 2 (c>a), but it is economically less viable in year n (d>b).

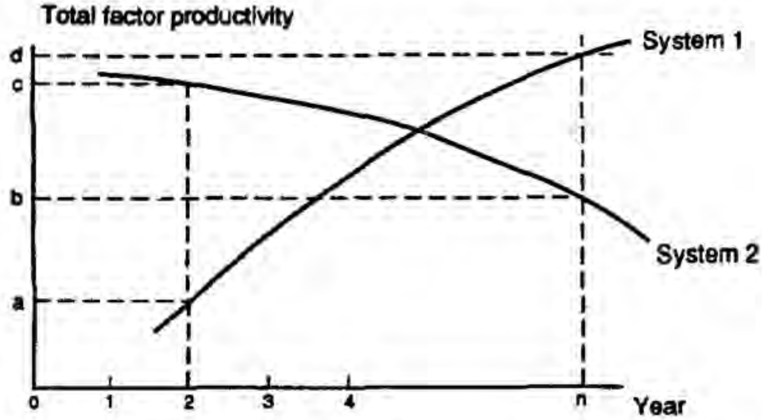

Figure 1. **Total factor productivity changes for two hypothetical agricultural systems over time.**

We next present the methodology for empirically measuring intertemporal and inter-spatial TFP. Readers not interested in the mathematical derivation of the models may go directly to section IV.

Indices of intertemporal and interspatial TFP

Assume that the agricultural production process of cropping system i in period t can be represented by the dual variable cost function:

$$G_{it} = G(Y_{it}, Z_{it}, W_{it}, B_{it}, T_t, D_i) \tag{11}$$

where G_{it} is the cost of production; W_{it} is a vector of input prices; Y_{it} is crop output; Z_{it} is the change in resource stock levels; B_{it} is the resource stock abundance level; and T_t and D_i denote the intertemporal and interspatial efficiency difference indicators, respectively. Derivation of the intertemporal and interspatial TFP indices depends critically on the proper specification of the total cost function C_{it}, which in turn depends on the nature of Z_{it} (i.e., whether the change in resource stock is positive or negative). We, therefore, consider two cases.

Case 1. Net positive change in resource stock: This represents the case where, for example, there is a net improvement in soil quality index. The resource flow is considered as an output, and in this case $C_{it} = G_{it}$. Assuming constant returns to scale and competitive markets, application of Diewert's (1976) quadratic lemma (see Annex B) to a logarithmic approximation of (11) gives:

$$\begin{aligned} \Delta LnC = {} & 1/2\,[R_{yis} + R_{yot}].[LnY_{is} - LnY_{ot}] \\ & + 1/2[R_{zis} + R_{zot}].[LnZ_{is} - LnZ_{ot}] \\ & + 1/2\Sigma k[S_{kis}+S_{kot}].[LnW_{kis}-LnW_{kot}] - [LnB_{is} - LnB_{ot}] + \theta_{io} + \mu_{st} \end{aligned} \tag{12}$$

where i and o represent two distinct cropping systems, and s and t represent two distinct time periods. S_{kis} and S_{kot} are the kth input factor cost shares; R_{yis} and R_{yot} are the revenue shares for product Y; and R_{zis} and R_{zot} are (implicit) revenue shares for resource flow Z. θ_{io} and μ_{st} denote the interspatial and intertemporal TFP effects and are defined as:

$$\theta_{io} = 1/2 \left[\frac{\partial LnG}{\partial D} \bigg|_{D=D_i} + \frac{\partial LnG}{\partial D} \bigg|_{D=D_o} \right] [D_i - D_o], \text{ and} \tag{13}$$

$$\mu_{st} = 1/2 \left[\frac{\partial LnG}{\partial T} \bigg|_{T=T_s} + \frac{\partial LnG}{\partial T} \bigg|_{T=T_t} \right] [Ts - Tt] \tag{14}$$

Equation (12) states that the cost difference across cropping systems and time periods can be broken into six terms, including: (1) an output effect, (2) a resource flow effect, (3) an input price effect, (4) a resource stock effect, (5) an interspatial effect, and (6) an intertemporal effect.

Following Denny and Fuss (1983), if we want to measure the intertemporal TFP (thus sustainability) of a particular technology, then $D_i = D_o = 0$. Solving for μ_{st} in (12) yields the dual measure of intertemporal productivity for two periods s and t.

$$\begin{aligned} \mu_{st} = [LnG_s - LnG_t] &- 1/2\, [R_{ys} + R_{yt}].[LnY_s - LnY_t] \\ &-1/2\, [R_{zs} + R_{zt}].[LnZ_s - LnZ_t] \\ &-1/2\, \Sigma_k [S_{ks} + S_{kt}].[LnW_{ks} - LnW_{kt}] \\ &+ [LnB_s - LnB_t] \end{aligned} \tag{15}$$

Similarly, the dual measure of interspatial productivity between system i and reference system o at a particular point in time ($T_s = T_t = o$) is

$$\begin{aligned} \theta_{io} = [LnG_i - LnG_o] &- 1/2\, [R_{yi} + R_{yo}].[LnY_i - LnY_o] \\ &- 1/2\, [R_{zi} + R_{zo}].[LnZ_i - LnZ_o] \\ &- 1/2\, \Sigma_k [S_{ki} + S_{ko}].[LnW_{ki} - LnW_{ko}] \\ &+ [LnB_i - LnB_o] \end{aligned} \tag{16}$$

Now turn to the primal space. Totally differentiating the log of the cost equation, $\dot{G} = \Sigma_i W_i X_i$, with respect to time, yields:

$$\dot{G} = \sum_i S_i \dot{X}_i + \sum_i S_i \dot{W}_i \tag{17}$$

The Tornqvist approximation to (17) for periods s and t and systems i and o gives:

$$\Delta LnG = [LnG_{is} - LnG_{ot}]$$

$$= 1/2\ \Sigma_k [S_{kis} + S_{kot}] . [LnX_{kis} - LnX_{kot}]$$

$$+ 1/2\ \Sigma_k [S_{kis} + S_{kot}] . [LnW_{kis} - LnW_{kot}] \qquad (18)$$

Equating (12) and (18) and solving for $-\mu_{st}$ and $-\theta_{io}$ gives measures of the intertemporal and interspatial productivity in the primal space.

$$\tau_{st} = -\mu_{st} = 1/2 [R_{ys} + R_{yt}].[LnY_s - LnY_t]$$

$$+ 1/2 [R_{zs} + R_{zt}].[LnZ_s - LnZ_t]$$

$$- 1/2\ \Sigma_k [S_{ks} + S_{kt}].[LnX_{ks} - LnX_{kt}]$$

$$-[LnB_s - LnB_t], \text{ and} \qquad (19)$$

$$\rho_{io} = -\theta_{io} = 1/2 [R_{yi} + R_{yo}].[LnY_i - LnY_o]$$

$$+ 1/2 [R_{zi} + R_{zo}].[LnZ_i - LnZ_o]$$

$$- 1/2\ \Sigma k [S_{ki} + S_{ko}].[LnX_{ki} - LnX_{ko}]$$
$$- [LnB_i - LnB_o] \qquad (20)$$

Note that under our assumptions, equations (19) and (20) are equal to the negative of the intertemporal and interspatial productivity measures that are obtained in the dual space (Ohta 1974).

Case 2. Net negative change in resource stock: This represents the case of, for example, a net depletion in soil nutrients. Here the farmer faces an implicit cost, which is not captured in conventional productivity measures and total cost $C_{it} = G_{it} + P_z Z$. Following the same procedure as in case 1, intertemporal and interspatial productivity measures, respectively, in the primal space are given by:

$$\tau_{st}' = [LnY_s - LnY_t] - 1/2 [S_{zs} + S_{zt}].[LnZ_s - LnZ_t]$$

$$-1/2\ \Sigma k [S_{ks} + S_{kt}].[LnX_{ks} - LnX_{kt}]$$
$$- [LnB_s - LnB_t], \text{ and} \qquad (21)$$

$$\rho_{io}' = [LnY_i - LnY_o] - 1/2 [S_{zi} + S_{zo}].[LnZ_i - LnZ_o]$$

$$-1/2\ \Sigma k [S_{ki} + S_{ko}].[LnX_{ki} - LnX_{ko}]$$

$$- [LnB_i - LnB_o] \qquad (22)$$

where S_{zs} and S_{zt} in equation (21) and S_{zi} and S_{zo} in equation (22) denote the (implicit) cost shares for depleted resource Z_t. The basic difference between τ_{st} and ρ_{io} and τ_{st}' and ρ_{io}' is that in the former the net increase in soil quality is treated as an output (benefit) while in the latter the decrease in the soil quality index is treated as a cost.

IV. An Empirical Application

In this section, we illustrate how the intertemporal and interspatial TFP measures developed in equations (19)-(22) can be used to determine the sustainability and economic viability of competing cropping systems. The analysis takes advantage of a set of data generated by a four-year (1985-1988) collaborative study between the United Nations University (UNU) and IITA on the effects of deforestation and land use on soil, hydrology, microclimate, and productivity in the humid coastal belt of Nigeria (Lal and Ghuman 1989).

Description of the cropping systems

We evaluated four cropping systems (denoted A, B, C, and D) over a two-year period (1986-1988) for which we had a complete and balanced set of data. In system A, land was cleared manually and cultivated by a traditional farmer. Crops grown in 1986 included yam, melon, and plantain. In 1988, plantain, melon, and cassava were grown, but cassava was not harvested. In all other systems, land was cleared mechanically with a tractor-mounted shear blade, and all trials were researcher-managed. In system B, cassava, maize, and cowpea were grown in 1986. In 1988 only cassava was planted. In system C, maize and cassava were grown in 1986 while only rice was grown in 1988. All crops in system C were grown in alleys formed by hedgerows of trees or shrubs. The trees or shrubs were periodically pruned during the cropping season to prevent shading and reduce competition with food crops (Kang et al. 1990). In system D, plantain was the only crop grown during the 1986-1988 period. No fertilizer was applied in any of the systems.

In order to calculate indices of sustainability and economic viability for the cropping systems, indices of output and input as well as of prices need first to be calculated. Table 1 presents the price and quantity data for the output, and Tables 2-5 present the quantity and price data for the inputs as well as the stock of natural resources. Tables 6 and 7 show the revenue shares, i.e., the proportion of total revenue contributed by aggregate crop output and changes in such nutrients (resource flow) as well as the cost shares, i.e., the proportion of total cost attributed to planting materials, labor, or implements and changes in soil nutrients (resource flow). Note that real prices, i.e., nominal prices adjusted for rate of inflation (at the rate of 5.05 in 1986 and 7.82 in 1988) were used in all calculations.

Valuation of Output

When more than one crop is produced in a year, an implicit quantity index of total output for the system is derived by dividing the total value of all crops harvested by an associated price index. The price index is obtained by weighting the price of each crop by its share in total revenue. Crop yields (tonnes/ha) for all cropping systems were obtained from Lal and Ghuman (1989). Farm level prices for most crops during 1986 and 1988 were obtained from unpublished reports of the Bendel State Agricultural Development Project (ADP). In a few cases, where crop prices were not available from Bendel ADP records, they were obtained from the neighboring Oyo State by IITA.

Table 1. Prices (Naira/t) and yields of crops (t/ha) for four cropping systems under experimental conditions in 1986 and 1988 in Bendel State, Nigeria.

	Nominal prices		Crop yields[1]							
	1986	1988	1986				1988			
Crops			A	B	C	D	A	B	C	D
Cassava	105	520	-	17.8	12.98	-	-	10.75	-	-
Rice	650	1270	-	-	-	-	-	-	0.16	-
Maize	580	1240	-	2.7	3.36	-	-	-	-	-
Cowpeas	2690	4840	-	0.4	-	-	-	-	-	-
Yams	710	1570	7.0	-	-	-	-	-	-	-
Plantains	490	1060	3.2	-	-	0.26	1.86	-	-	0.8
Melon	2620	3820	0.07	-	-	-	0.2	-	-	-

1. A, B, C, and D denote different crop combinations (see text).

Table 2. Price (Naira/unit)and quantity (per ha) of inputs used in System A, Bendel State (Nigeria) during 1986 and 1988.

	Nominal price		Quantity		
Inputs	1986	1988	Unit[1]	1986	1988
Planting materials					
Yams	0.71	-	kg	600	-
Plantains	0.35	0.75	S	173	1730
Melon	2.62	3.50	kg	15	15
Labor	6	9	MD	312	237
Implements					
Machetes	25	35	No.	2.64	2.64
Hoes	10	20	No.	2.44	2.44
Total soil nutrients in 0-10cm soil[2]					
Nitrogen	0.7308	0.7523	kg	1549.0	1609.0
Phosphorus	0.2912	0.3034	kg	14.7	15.2
Potassium	0.2213	0.2352	kg	50.0	83.1

1. S = suckers, MD = man-days. 2. At standard bulk density of $1.21 g/cm^3$.

Table 3. Price (Naira/unit) and quantity (per ha) of inputs used in system B, Bendel State, Nigeria during 1986 and 1988.

Input	Nominal price		Quantity		
	1986	1988	Unit[1]	1986	1988
Planting materials					
Cassava	3	5	b	60	60
Maize	0.5	-	kg	30	-
Cowpeas	3	-	kg	25	-
Labor	6	9	MD	190	44
Implements					
Machetes	25	35	No.	2.64	2.64
Hoes	10	20	No.	2.44	2.44
Total soil nutrients in 0-10cm soil[2]					
Nitrogen	0.7308	0.7523	kg	1355.2	1355.2
Phosphorus	0.2912	0.3034	kg	17.06	13.43
Potassium	0.2213	0.2352	kg	139.66	69.84

1. b = bundle, MD = man-days. 2. At standard bulk density of 1.21 g/cm^3.

Table 4. Price (Naira/unit) and quantity (per ha) of inputs used in system C, Bendel State, Nigeria during 1986 and 1988.

Input	Nominal price		Quantity		
	1986	1988	Unit[1]	1986	1988
Planting Materials					
Maize	0.5	-	kg	30	-
Cassava	3	-	b	50	-
Rice	-	1.27	kg	-	60
Labor	6	9	MD	108.5	122
Implements					
Machetes	25	35	No.	2.64	2.64
Hoes	10	20	No.	2.44	2.44
Total soil nutrients in 0-10cm soil[2]					
Nitrogen	0.7308	0.7523	kg	1258.4	1645.6
Phosphorus	0.2912	0.3034	kg	14.08	10.04
Potassium	0.2213	0.2352	kg	90.6	93.4

1. b = bundle, MD = man-days. 2. At standard bulk density of 1.21 g/cm^3.

Table 5. Price (Naira/unit) and quantity (per ha) of inputs used in system D, Bendel State, Nigeria, during 1986 and 1988.

Input	Nominal price 1986	Nominal price 1988	Quantity Unit[1]	Quantity 1986	Quantity 1988
Planting materials					
Plantains	0.4	-	S	2200	2200
Labor	6	9	MD	33	22
Implements					
Machetes	20	35	No.	2.64	2.64
Hoes	10	20	No.	2.44	2.44
Total soil nutrients in 0-10cm soil[2]					
Nitrogen	0.7308	0.7523	kg	1645.6	1282.6
Phosphorus	0.2912	0.3034	kg	13.04	13.91
Potassium	0.2213	0.2352	kg	78.3	100.1

1. S=suckers, MD = man-days 2. At standard bulk density of 1.21 g/cm^3.

Table 6. Revenue and cost shares used in the computation of intertemporal total factor productivity (sustainability) indices (1986-1988).

Systems	Revenue share: Crop output	Revenue share: Resource flow	Cost share: Materials	Cost share: Labor	Cost share: Implements	Cost share: Resource flow
A+	0.986	.014	0.383	0.605	0.012	0.00
B+	0.998	.002	0.407	0.425	0.055	0.00
C+	0.402	.598	0.063	0.904	0.034	0.00
D++	1.000	0.000	0.713	0.107	0.022	0.158

1. One plus (+) indicates the case of a net positive change in resource abundance. Two pluses (++) indicate the case of a net negative change in resource abundance level.

Table 7. Revenue and cost shares used in the computation of interspatial total factor productivity (economic viability) indices in 1986 and 1988.

Systems	Revenue share		Cost share			
	Crop output	Resource flow	Materials	Labor	Implements	Resource flow
System A						
1986++	1.000	0.000	0.360	0.630	0.009	0.000
1988++	1.000	0.000	0.383	0.605	0.120	0.000
System B						
1986++	1.000	0.000	0.168	0.709	0.0171	0.105
1988++	1.000	0.000	0.322	0.425	0.0440	0.209
System C						
1986++	1.000	0.000	0.153	0.606	0.0260	0.215
1988++	0.880	0.120	0.063	0.904	0.0340	0.000
System D						
1986+	0.653	0.347	0.796	0.179	0.0250	0.000
1988++	1.000	0.000	0.725	0.109	0.0220	0.144

1. One plus (+) indicates the case of a net positive change in resource abundance. Two pluses (++) indicate the case of a net negative change in resource abundance level.

Valuation of inputs

An aggregate input index was calculated, using quantities of labor, planting materials, and implements. Labor in man days per hectare for all operations was taken from data collected during the experiments. We used the average daily wage rate for each year, and assumed for simplicity that family workers are valued at their opportunity cost and hence paid at the same wage as hired workers. An implicit index of planting materials (seeds and suckers) was computed as the ratio of total expenditure on these materials to the material input price index. The latter was computed by weighting the price of each input by its share in total cost of material inputs. Quantities were those used in the experiments, while prices were either those recorded or imputed using secondary data from the Bendel State ADP or IITA.

An index for the quantity of implements was computed as the ratio of total annual cost of capital inputs to the capital service price. To create an aggregate capital service price, we share-weighted the price of each farm tool in the same manner as for the aggregate material price index. Annual cost of capital was defined as the sum of the annual user costs for all implements. These were calculated using the capital recovery factor formula, $A=PV\ [r/1-(1+r)^{-t}]$, where A is the annualized cost of the capital item; PV is the present value of the capital item defined as the purchase price less the present worth of its future salvage value; t is the estimated life span of the capital item; and r is the discount rate. A discount of 10 percent was used. This is the opportunity cost of capital for Nigeria (IBRD 1987).

Valuation of natural resource stocks and flows

In this example, only changes in the major soil nutrients, nitrogen (N), phosphorus (P), and potassium (K), are considered. Changes in their natural abundance levels are accounted for by aggregating the three soil nutrients. To construct the Divisia index for the soil resource stock, we share-weight the quantities of N, P, and K (kg/ha) available in the top 0-10cm soil. To obtain the share weight, we approximated the opportunity cost of each soil nutrient by its replacement cost, i.e., the market price of each element in commercial fertilizers. Resource flows are the difference between resource abundance levels between 1986 and 1988 for each cropping system or in two cropping systems in a given year. Per hectare quantities of soil N, P, and K were computed using an average bulk density level of 1.21 g/cm^3. Fertilizer prices during 1986 and 1988 were obtained by consulting unpublished reports of the Bendel State ADP. The price of N was computed as the average of nitrogen prices in urea (U), diammonium phosphate (DAP), calcium ammonium nitrate (CAN), and ammonium sulfate (AS). We assume that the price of N in DAP, CAN, and AS is proportional to the nitrogen content in each product. The prices of P and K are calculated in the same manner as that of N, using DAP and single superphosphate (SSP) for phosphorus and muriate of potash (MOP) for potassium. Note that, in Nigeria, fertilizer prices are set by the government and are, therefore, uniform across the country. In this example, the subsidized fertilizer prices were used. Unsubsidized border equivalent prices could also be computed and used.

Empirical results

Intertemporal (sustainability) and interspatial (economic viability) indices for the four cropping systems were calculated using equations (19)-(22). Results are reported in Tables 8 and 9. Columns I-III illustrate the potential biases that may result when common property resource stocks and flows are not properly accounted for. In column I, there is no adjustment for changes in resource stock abundance levels and flows. Column II shows the resulting indices when we take into consideration only changes in stock levels. In column III, changes in both the resource stock levels and the flows are taken into consideration.

After accounting for resource stock and flows, column III in Table 8 indicates that between 1986 and 1988, TFP increased for systems B and C (index > 1) and declined for systems A and D (index < 1). Systems B and C produced 6.25 and 11.58 times as much output in 1988 as in 1986, using the 1986 input bundle as the base. Systems B and C are, therefore, sustainable since after fully accounting for temporal differences in input quality and quantity and resource flows and stocks, they produce more output than in the reference year. Using a similar accounting procedure, systems A and D produced only 0.22 and 0.88 as much output in 1988 as in 1986. Thus, A and D are nonsustainable.

Note from Table 8 that accounting for changes in natural resource abundance levels as well as flows substantially alters productivity measures. This is particularly true for system C, in which the associated shrubs fix atmospheric nitrogen and recycle nutrients, and system D, where plantains deplete the soil of its nutrients' stock. Note in system C that if we do not take into consideration the net addition to soil nitrogen, the sustainability index is lower than unity (column I), erroneously indicating that the system is not sustainable. Soil nutrients increased by 31 percent, representing nearly 60 percent of the gross revenue in 1988 (Table 6). This is an important contribution to the value of output, which explains the high intertemporal productivity index number in system C.

Table 8. Indices of intertemporal total factor productivity (sustainability) for four cropping systems in Bendel State, Nigeria, 1986-1988[1].

Systems[2]	Intertemporal TFP indices[3]		
	I[3]	II	III
A	0.20	0.19+	0.22+
B	6.38	6.14+	6.25+
C	0.02	0.01+	11.58+
D	3.27	4.23++	0.88++

1. Numbers with one plus (+) indicate a net positive change in resource abundance (total value of natural resources) while those with two pluses (++) indicate a net negative change in resource abundance levels.
2. Refer to text for details of the various systems.
3. In column I there is no correction for changes in soil resource stock and flows. Column II accounts for changes in resource stocks only. Column III accounts for changes in both resource stocks and flows.

Similarly in system D, if we do not account for the depletion of soil resources, the sustainability index is greater than unity (columns I and II), erroneously indicating that the system is sustainable. The stock of soil nutrients in this system decreased by 23 percent, representing about 16 percent of the total cost of production (Table 6). Because it is treated as a technological constraint, an increase (decrease) in natural resource stock level at a point in time serves to lower (increase) the rate of growth of productivity. However, increases or decreases in resource abundance levels are themselves associated with increases or decreases in resource flows that affect the rate of growth of productivity. The extent to which productivity levels are affected depends on the share of the resource flow in total revenue or cost. Note that in systems A and B, TFP measures do not change much between columns I and III. Although the stock of soil nutrients increased over time, the increases represent only 1.4 and 0.2 percent of the total net revenue in the systems as shown in Table 6.

Those results clearly indicate that unless variations in the resource flows and natural resource stock abundance levels are accounted for, the inferences will be biased. The bias will depend on the magnitude of the resource flow and stock. A positive change in the resource stock level is of potential benefit to the farmer and society and thus contributes to improving the sustainability of the system. When the change in the natural resource stock is negative, the farmer and society face a cost (though it is hidden) that negatively affects the system's sustainability.

Table 9 presents the indices of economic viability for cropping systems B, C, and D relative to A in 1986 and 1988. In 1986, after accounting for changes in resource abundance and flows during the year (column III), systems B and C are shown to be relatively less productive than the reference system A. The economic viability (interspatial TFP) indices are estimated to be 0.73 for system B and 0.76 for system C, respectively, indicating that these systems use relatively more resources and produce a comparatively lower output than system A. Only system D, in which plantain was grown, is more productive than system A. The interspatial TFP index is estimated to be 2.40, indicating that system D produces more output than system A after accounting for differences in input quantity and resource stock and flow changes.

Table 9. Indices of interspatial total factor productivity (economic viability) for four cropping systems in Bendel State, Nigeria, during 1986 and 1988[1].

Systems[2]	Interspatial TFP indices[3]					
	1986			1988		
	I	II	III	I	II	III
A	1	1	1	1	1	1
B	1.73	2.02++	0.73++	68.50	81.34++	9.26++
C	5.37	6.68++	0.76++	0.37	0.36+	1.12+
D	0.06	0.18+	2.40+	1.04	1.31+	0.14++

1. Numbers with one plus (+) indicate a net positive change in resource abundance (total value of natural resources) while those with two pluses (++) indicate a net negative change in resource abundance levels.
2. Refer to text for details of the various systems.
3. In column I there is no correction for changes in soil resource stock and flows. Column II accounts for changes in resource stocks only. Column III accounts for changes in both resource stocks and flows.

In 1988, productivity indices for all the systems show a different pattern. With interspatial TFP indices of 9.26 and 1.12, systems B and C are more economically viable than system A. Similarly, with a TFP index of 0.14, system D is found to be less economically viable than system A. The changes in relative productivity of the systems in 1988 compared with 1986 are attributable to soil nutrient changes over the two-year period. In system C, where crops are grown in association with leguminous trees, benefits are obtained in the form of increased crop output and improvement in the soil quality index. In 1988, the stock of soil nutrients increased by 2.3 percent compared system A, representing about 12 percent of the gross revenue in system C (Table 7).

In the plantain system D, soil fertility was depleted. The stock of soil nutrients decreased by 21 percent in this system compared with system A, representing about 14 percent of total cost (Table 7). We note from columns I and II in Table 9 that, as in Table 8, when variations in resource stock levels and flows are not accounted for, productivity measures are biased.

V. Conclusions and Suggestions for Further Research

New technologies must be developed in SSA, which are sustainable and economically viable. Despite widespread concern about sustainability of agricultural systems, there is little guidance in the literature on what practical methods are to be used for measuring the sustainability and economic viability of a cropping system. In this paper, a model for measuring agricultural sustainability and economic viability is presented. The model is based on the concept of total factor productivity, defined as the ratio of aggregate output to aggregate input, and the growth accounting procedure. Because agriculture is a sector which utilizes common property resources (e.g., soil nutrients), conventional measures of total factor productivity are inappropriate. A general model of TFP measurement is, therefore, developed, which takes into account natural resource inputs and outputs.

To measure "economic viability" and "sustainability", we advocate the interspatial and intertemporal TFP measures of Denny and Fuss (1983). Interspatial TFP measures the economic viability of one system relative to another at a given time (e.g., one crop season), and it is technically defined as the logarithm difference in the indices of the value of outputs of the two production systems minus the logarithm difference in the indices of the value of their inputs (including conventional and natural resource inputs and outputs). Thus, system X is said to be economically more viable than system Y if, after accounting for spatial differences in input levels as well as natural resource stocks and flows, X produces more output than Y.

Similarly, intertemporal TFP, which measures the sustainability of a given cropping system, is defined as the rate of change of an index of outputs divided by an index of inputs (including conventional inputs and natural resource stock and flows). A production system will be said to be sustainable over time if, after fully accounting for differences in factor inputs and natural resource stocks and flows, it produces at least the same amount of output as previously. The intertemporal and interspatial productivity measures can be computed using the growth accounting method and economic index numbers, thus eliminating the need for use of econometric estimation. To use the measures, however, all output and input prices must be known.

Using a unique set of experimental data available at the International Institute of Tropical Agriculture, intertemporal and interspatial TFP indices were computed for four experimental cropping systems at a site in Bendel State, Nigeria. The results show that sustainability and economic viability indices are sensitive to the incorporation of natural resource stocks and flows. Where changes in soil nutrient status are important, the productivity measures provide markedly different results from conventional TFP approaches.

In this paper, only changes in soil nutrient status are taken into consideration. The model needs to be expanded to take into consideration changes in other common property resources. The appropriate set of such resources needs to be established by agricultural scientists researching the sustainability of agricultural systems. For example, the following data is being collected in a long-term experiment at IITA:

- vegetation; weed density, composition of flora, and weed seed dynamics.
- physical properties: bulk density, soil water retention, water infiltration rate, particle size distribution.

- soil chemical properties: organic C, N, P, and pH and cation exchangeable capacity (CEC).
- soil microbiological aspects: soil organic matter fractions (heavy and light fractions, including microbial biomass).
- agronomic factors: plant establishment, plant biomass, crop yields.
- environmetal factors: rainfall, light interception, groundwater pollution.
- economic factors: labor use, input cost, and output prices.

Obviously, for other systems, other minimum data sets are needed, based on our further understanding of the processes and determinants of sustainability.

In addition to establishing the appropriate minimum set of variables to be measured when the sustainability of cropping systems are evaluated, scientists also need to quantify the relationship between crop production and changes in the levels of the variables being investigated. Relationships such as the effect of changes in soil bulk density, weed infestation, insect infestation, etc., on crop yield in given cropping systems, once they are established, can be incorporated into the sustainability and economic viability indices.

Agricultural scientists also need to establish the appropriate time period over which the sustainability of a cropping system should be evaluated. The indices proposed here can be used over short as well as long time periods. Long-term experiments need to be established, which allow sufficient time for observation of trends in the variables being observed.

Finally, the prices used in valuation of inputs and outputs, including natural resources, must reflect the true value of the resources to society. Computation of the appropriate social accounting, or shadow prices, is a major challenge to social scientists.

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Annex A

Development of Continous Time Divisia Index

Constant returns to scale assumption implies that:

$$\partial LnG/\partial LnY + \partial LnG/\partial LnZ = 1 \tag{A1}$$

Using the first order conditions of problem (8), (A1) can be re-expressed as:

$$P_yY + P_ZZ = G = C \tag{A2}$$

Totally differentiating the log of total cost C with respect to t yields:

$$\begin{aligned}\dot{C} &= [\partial G/\partial t + Gy(dY/dt) + G_Z(dZ/dt)] \\ &\quad + \Sigma jG_{wj}(dWj/dt) + G_B(dB/dt)]/C\end{aligned} \tag{A3}$$

The Divisia index of productivity is obtained by deriving $\dot{C}$ directly from total cost, $C = \Sigma jWjXj$:

$$\dot{C} = [\Sigma jWjXj\dot{W}j + \Sigma jWjXj\dot{X}j]/C \tag{A4}$$

Equating (A11) and (A12) and solving for $-[\partial G/\partial t]/C$ gives (after rearranging):

$$\begin{aligned}-[\partial G/\partial t]/C &= [GyY/C]\dot{Y} + [G_ZZ/C]\dot{Z} \\ &\quad -\Sigma j(WjXj/C)\dot{X}j+[G_BB/C]\dot{B}\end{aligned} \tag{A5}$$

Using the first order conditions of problem (8), (A5) can be re-expressed as:

$$\begin{aligned}-\partial LnC/\partial t &= [P_yY/C]\dot{Y} + [P_ZZ/C]\dot{Z} \\ &\quad - \Sigma j(WjXj/C)\dot{X}j - \dot{B}\end{aligned} \tag{A6}$$

where for the open access situation, $C_B = G_B$ and $G_B(B/C)=-1$ for a Schaefer type of technology (see Squires and Herrick 1990 for proof).

In the case that changes in resource abundance levels are negative, constant returns to scale assumption implies that:

$$P_yY = G(\bullet) + P_ZZ \quad = C \tag{A7}$$

Following the same procedure, as above, the Divisia index of total factor productivity is given by:

$$\begin{aligned}-\partial LnC/\partial t &= [P_yY/C]\dot{Y} - [P_ZZ/C]\dot{Z} \\ &\quad - \Sigma j(WjXj/C)\dot{X}j - \dot{B}\end{aligned} \tag{A8}$$

Annex B

Derivation of Cost Function

Diewert's (1976) quadratic lemma basically states that, if a function is quadratic, the difference between the function's values evaluated at two points is equal to the average of the gradient evaluated at both points multiplied by the difference between the points:

$$f(Z^1)-f(Z^0) = 1/2[f(Z^1)+f(Z^0)]^T(Z^1-Z^0) \qquad \text{(B1)}$$

where $f(Z^r)$ is the gradient vector of f evaluated at Z^r, r=0,1.

Applying Diewert's (1976) quadratic lemma to a logarithmic approximation of the cost function $G_{it} = G(Y_{it}, Z_{it}, W_{it}, B_{it}, T_t, D_i)$ gives:

$$\Delta LnC = LnC_{is} - LnC_{ot}$$

$$= 1/2\left[\frac{\partial LnG}{\partial LnY}\Bigg|_{Y=Y_{is}} + \frac{\partial LnG}{\partial LnY}\Bigg|_{Y=Y_{ot}}\right].[LnYis-LnYot]$$

$$+ 1/2\left[\frac{\partial LnG}{\partial LnZ}\Bigg|_{Z=Z_{is}} + \frac{\partial LnG}{\partial LnZ}\Bigg|_{Z=Z_{ot}}\right].[LnZ_{is}-LnZ_{ot}]$$

$$+1/2\left[\Sigma_k \frac{\partial LnG}{\partial LnW_k}\Bigg|_{W_k=W_{is}} + \frac{\partial LnG}{\partial LnW_k}\Bigg|_{W_k=W_{kot}}\right].[LnW_{kis}-LnW_{kot}]$$

$$+ 1/2\left[\frac{\partial LnG}{\partial LnB}\Bigg|_{B=B_{is}} + \frac{\partial LnG}{\partial LnB}\Bigg|_{B=B_{ot}}\right].[LnB_{is}-LnB_{ot}]$$

$$+ 1/2\left[\frac{\partial LnG}{\partial D}\Bigg|_{D=D_i} + \frac{\partial LnG}{\partial D}\Bigg|_{D=D_o}\right].[D_i-D_o]$$

$$+ 1/2\left[\frac{\partial LnG}{\partial T}\Bigg|_{T=T_s} + \frac{\partial LnG}{\partial T}\Bigg|_{T=T_t}\right].[T_s-T_t] \qquad \text{(B2)}$$

where i and o represent two distinct cropping (or land use) systems, and s and t represent two distinct time periods.

Assuming contant returns to scale and competitive factor markets, we can rewrite equation (B2) as:

$$
\begin{aligned}
\Delta LnC = {} & 1/2\,[R_{yis} + R_{yot}].[LnY_{is} - LnY_{ot}] \\
& + 1/2[R_{zis} + R_{zot}].[LnZ_{is} - LnZ_{ot}] \\
& + 1/2\Sigma_k\,[S_{kis} + S_{kot}].[LnW_{kis} - LnW_{kot}] \\
& - [LnB_{is} - LnB_{ot}] + \theta_{io} + \mu_{st}
\end{aligned} \tag{B3}
$$

where θio and μst denote the interspatial and intertemporal effects as:

$$
\theta io = 1/2 \left[\frac{\partial LnG}{\partial D} \bigg|_{D=D_i} + \frac{\partial LnG}{\partial D} \bigg|_{D=D_o} \right] [D_i - D_o] \tag{B4}
$$

and the intertemporal effect as

$$
\mu st = 1/2 \left[\frac{\partial LnG}{\partial T} \bigg|_{T=T_s} + \frac{\partial LnG}{\partial T} \bigg|_{T=T_t} \right] [T_s - T_t] \tag{B5}
$$

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